LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIRST SEMESTER – NOVEMBER 2018

16/17/18UMT1AL02 – MATHEMATICS FOR STATISTICS - I

Date: 02-11-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART A

Answer all the questions: **(10 X 2 = 20)**

1. Define diagonal matrix.
2. If $A = \begin{pmatrix} 1+i & 2+3i & 2 \\ 3-4i & 4+5i & 1 \\ 5 & 3 & 3-i \end{pmatrix}$, find conjugate transpose matrix A^*.
3. Define characteristic equation.
4. Find the sum of the eigen values of the matrix $\begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{pmatrix}$.
5. If $y = 4x^3 - 2x + \frac{3}{x^3}$, find $\frac{dy}{dx}$.
6. Differentiate e^t with respect to $\sqrt{t}$.
7. For what values of x is $2x^3 - 9x^2 + 12x + 4$ a decreasing function?
8. Find the points of inflexion in the curve $y = x^3 - 9x^2 + 7x - 6$.
9. Evaluate $\int \left(x + \frac{1}{x} + e^x\right) dx$.
10. State any two properties of definite integrals.

PART B

Answer any FIVE questions: **(5 X 8 = 40)**

11. Show that the matrix $B = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is orthogonal.
12. Find the inverse of the matrix $D = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$.
13. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.
14. (a) If $y = x^{x^{x^{\cdots^{\infty}}}}$, find $\frac{dy}{dx}$.

    (b) If $x^y = y^x$, prove that $\frac{dy}{dx} = \frac{y(y - x\log y)}{x(x - y\log x)}$. (4+4)
15. Show that for $x > 0$, $x - \frac{1}{2}x^2 < log(1+x) < x$.
16. Find the n^{th} differential coefficient of $x^3 log\, x$.
17. Evaluate $\int \frac{2x+3}{x^2+x+1} dx$.
18. Prove that $\int_0^{\frac{\pi}{2}} \frac{(sin\, x)^{\frac{3}{2}}}{(sin\, x)^{\frac{3}{2}} + (cos\, x)^{\frac{3}{2}}} dx = \frac{\pi}{4}$.

PART C

Answer any TWO questions: **(2 X 20 = 40)**

19. (a) Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+ac+bc)$.

(b) Solve $6x + y - 3z - 5 = 0;\ \ x + 3y - 2z - 5 = 0;\ \ 2x + y + 4z - 8 = 0$ using Cramer's rule. (10+10)

20. (a) Find the characteristic roots and associated characteristic vectors of the matrix

$$A = \begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}.$$

(b) If $(sin\, x)^{cos\, y} = (sin\, y)^{cos\, x}$, find $\frac{dy}{dx}$. (15+5)

21. (a) If $u = log\left(\frac{x^2+y^2}{xy}\right)$, show that $\frac{\partial^2 u}{\partial x\, \partial y} = \frac{\partial^2 u}{\partial y\, \partial x}$.

(b) Find the maximum and minimum values of the function $u = 2(x^2 - y^2) - x^4 + y^4$. (5+15)

22. (a) Evaluate $\int (log\, x)^2 dx$.

(b) Integrate $\frac{x}{(x-1)(x-2)(x-3)}$ with respect x. (8+12)

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